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Scale anomaly induced instanton interaction[1]

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Abstract

The binary interaction of large size instantons in a SU(2) Yang-Mills theory is obtained from the one-loop effective action for the field strength. The instanton interaction is calculated as a function of the instanton separation and in dependence on radius and relative orientation of the instantons. Two equally oriented instantons with radii large compared with the scale defined by the gluon condensate have purely attractive interaction, whereas the interaction of maximal disoriented instantons is repulsive. We argue that the medium range attractive interaction of the instantons generally holds and is solely due to the instability of the perturbative vacuum.

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1. Introduction

The confidence in QCD as the correct theory of strong interactions stems from the excellent agreement between QCD predictions and high energy scattering experiments [1]. This success, from a theoretical point of view, is due to QCD's remarkable property of asymptotic freedom [1], which implies that physics, involving high momentum transfers, can be described in a perturbative expansion with respect to the coupling. At low energy, however, the running coupling constant is expected to be large implying that a study of the QCD ground state is beyond the scope of perturbation theory. For this reason only a few aspects of the QCD vacuum are known up to now. An important property is that scale invariance of pure Yang-Mills theory is anomalously broken by quantum fluctuations [2] indicated by a non-vanishing value for the gluon condensate. In this context instantons [3] play an important role for describing the QCD vacuum, since they give rise to a gluon condensate. Furthermore instantons may possibly trigger spontaneous breaking of chiral symmetry [4] and offer an explanation of the $U_A(1)$ problem [5].

Instantons are gauge field configurations which minimise the euclidean Yang Mills action and correspond to localised spots of self- or antiself-dual field strength. Investigations of the interaction between instantons, originating from the classical Yang-Mills action, show that the gluonic vacuum is not realised as a dilute gas of instantons [4], but rather than an instanton liquid [6, 7, 8]. Recent investigations which include shape variations of the instantons indicate that instantons might lose their identity in a strongly correlated instanton medium [9].

Instantons have a free scale parameter, the instanton radius, which reflects the scale invariance of the classical Yang Mills action. If the effects of fluctuations around the instanton [10] are studied, a second scale provided by the scale anomaly [2] enters. Calculating the one-loop effective action in dependence on the instanton radius it was observed that the effective action of a single instanton is not bounded from below for large instanton radii [10] indicating a infrared instability. In order to investigate the infared behaviour of instantons in a instanton medium the interaction of instantons induced by quantum fluctuations is needed for large instanton radii.

In this letter we study this interaction, provided by the effective action, of two widely separated instantons with large instanton radii. The interaction is studied in dependence on the two scales provided by the instanton radius and the scale set by the trace anomaly, i.e. the gluon condensate. We will find that the instanton interaction is purely attractive for instantons with radii large compared with the scale given by the gluon condensate. We argue that the attractive interaction at medium range does not depend on the details of the effective potential, but is due to the instability of the perturbative vacuum.

2. Scale anomaly and effective action

The classical Yang-Mills action is invariant under scale transformations giving rise to a conserved Noether current j_μ which can be related to the trace of the energy momentum tensor [2], $\partial^\mu j_\mu = \Theta^\mu_\mu = 0$. In the pioneering work of Collins et al. [2] it was shown that the scale invariance is broken at quantum level implying a non-vanishing vacuum expectation value of the energy momentum tensor, i.e.

$$\left\langle \Theta^\mu_\mu \right\rangle \;=\; \frac{\beta(g)}{2g^3} \left\langle F^a_{\mu\nu} F^a_{\mu\nu} \right\rangle \,, \tag{1}$$

where g is the renormalised coupling strength, $\beta(g)$ is the renormalisation group β-function and $F^a_{\mu\nu}$ is the field strength tensor. At one loop level we have $\beta(g) \approx -\beta_0 g^3$ with $\beta_0 = 11N/48\pi^2$ for a SU(N) gauge group. Therefore (1) implies in this case that $F^2 = \left\langle F^a_{\mu\nu} F^a_{\mu\nu} \right\rangle$ coincides with $\left\langle \Theta^\mu_\mu \right\rangle$, a physical quantity, and hence must be renormalisation group invariant at one loop level.

The effective potential for constant chromomagnetic fields was first calculated by Savidy [11]. He found that the perturbative vacuum is unstable, since the global minimum of the effective potential occurs at non-zero field $H = H_{min}$. It was then pointed out by P. Olesen [12] that the loop-expansion breaks down for fields $H \approx H_{min}$. Furthermore it was argued [13] that this breakdown for large fields $H \approx H_{min}$ is due to an instability of the Savidy vacuum against the formation of domains of constant magnetic field strength. For small fields the loop-expansion is reliable. We will find (see section 4) that the medium range instanton interaction is solely induced by the effective potential at small field strength, where Savidy's potential can be used.

A gauge and Lorentz covariant generalisation of the one-loop potential obtained in [11] to constant field strength $F^a_{\mu\nu}$ is given by

$$V(F^2) \;=\; \frac{1}{4g^2(\mu)} F^a_{\mu\nu} F^a_{\mu\nu} \;+\; \frac{\beta_0}{8} F^a_{\mu\nu} F^a_{\mu\nu} \, \ln \, F^a_{\mu\nu} F^a_{\mu\nu} / \mu^4 \,, \tag{2}$$

where μ is the renormalisation point. It is easily seen that this effective potential reflects the anomalous breaking of scale invariance. Consider for this purpose the scale transformation $V(F^2) \to V_\lambda = e^{4\lambda} V(e^{-4\lambda} F^2)$. One readily verifies the correct scale anomaly at one loop level from (2)

$$\partial^\mu j_\mu \;=\; \frac{d}{d\lambda} V_\lambda |_{\lambda=0} \;=\; -\frac{\beta_0}{2} F^2 \,. \tag{3}$$

Let us now show that the effective potential (2) follows in fact from very general arguments. First consider the renormalisation group equation

$$\mu \frac{dV}{d\mu} \;=\; \left(\mu \frac{\partial}{\partial \mu} \;+\; \beta(g) \frac{\partial}{\partial g} \right) \, V(F^2, \mu, g) \;=\; 0 \,, \tag{4}$$

where the definition $\beta(g) = \mu \frac{dg(\mu)}{d\mu}$ and the renormalisation group invariance of the field strength tensor at one loop level, $\frac{d}{d\mu} F^2 = 0$, was used. From dimensional arguments we have $V(F^2, \mu, g) = \mu^4 f(F^2/\mu^4, g)$, and (4) can be rewritten as a partial differential equation determining the F^2 dependence of V, i.e.

$$F^2 \frac{\partial V}{\partial F^2} - \frac{\beta(g)}{4} \frac{\partial V}{\partial g} - V = 0 . \tag{5}$$

Using the boundary conditions [2] $\frac{\partial V}{\partial(1/4g^2)} = F^2$ and $\lim_{F^2 \to 0} V = 0$ the solution to (5) is precisely given by (2). Renormalisation group invariance (4) also guarantees that the effective potential V does not depend on the arbitrary subtraction point μ. For further investigations it is convenient to rewrite V in an explicitly μ-independent form. For this purpose we introduce the gluon condensate F_0^2 which minimises the effective potential $V(F^2)$, i.e.

$$\frac{dV}{dF^2}|_{F_0^2} = \frac{1}{4g^2} + \frac{\beta_0}{8} \left(\ln F_0^2/\mu^4 + 1 \right) = 0 . \tag{6}$$

Eliminating μ in the effective potential V (2) in favour of F_0^2 with the help of (6) we obtain

$$V(F^2) = \frac{\beta_0}{8} F^2 \left(\ln \frac{F^2}{F_0^2} - 1 \right) . \tag{7}$$

This is the desired result because V neither depend on the renormalisation point μ nor on the renormalised coupling g.
In order to estimate the interaction between large size instantons the effective action $\Gamma[F^a_{\mu\nu}(x)]$ is needed. In leading order derivative expansion this effective action is given by

$$\Gamma[F^a_{\mu\nu}(x)] = \int d^4x \, V \left(F^2(x) \right) \tag{8}$$

where gradients on $F^a_{\mu\nu}(x)$, which occur in gauge invariant combinations of the covariant derivative D^{ab}_μ and $F^a_{\mu\nu}(x)$, are omitted. For an instanton medium these gradients have the order of magnitude $1/\rho$ with ρ being the instanton radius. This scale is to be compared the with gluon condensate implying that the terms omitted in (8) are of order $1/F_0^2\rho^4$ and are suppressed for large instantons.
We use the effective action (8) to estimate the static interaction between instantons in SU(N) Yang-Mills theories. We shall confine ourselves to the SU(N) instantons constructed in reference [14].

3. Binary instanton interaction

We define the binary (anti-) instanton interaction Γ_I and the interaction of an instanton anti-instanton pair, $\Gamma_{\bar{I}}$, by

$$\Gamma_{I/\bar{I}} := \Gamma[F^{(2)\,a}_{\mu\nu}(x)] - 2\,\Gamma[F^{(1)\,a}_{\mu\nu}(x)] , \tag{9}$$

where $F^{(2)\,k}_{\mu\nu}$ is the field strength configuration of two (anti-) instantons or of an instanton anti-instanton pair, respectively, and $F^{(1)\,k}_{\mu\nu}$ denote the field strengths of a single instanton. For definiteness we will consider the binary interaction of a definite type of instantons, obtained in [14]. For these instantons the gauge potential and the corresponding field strength is given by

$$A^a_\mu(x) \;=\; G^a_i \eta^i_{\mu\nu}\, x_\nu \, \frac{2}{x^2+\rho^2} \, , \tag{10}$$

$$F^a_{\mu\nu}(x) \;=\; G^a_i \eta^i_{\mu\nu}\, \psi(x^2) \, , \qquad \psi(x^2) := -\frac{4\rho^2}{(x^2+\rho^2)^2} \, , \tag{11}$$

where $\eta^i_{\mu\nu}$, $i = 1 \ldots 3$ are the self-dual 't Hooft matrices and ρ the instanton radius. Anti-instantons are provided by (10,11) by replacing $\eta^i_{\mu\nu}$ by the antiself-dual 't Hooft matrices $\bar\eta^i_{\mu\nu}$. Furthermore, the SU(N) valued matrices $G_i = G^a_i t^a$ (t^a generators of the SU(N) group) have to fulfill an SU(2) algebra [14]

$$[G_i, G_k] \;=\; i\epsilon_{ikl}\, G_l \, . \tag{12}$$

For $G^a_i = \delta_{ai}$, $a = 1 \ldots 3$ and $G^a_i = 0$, $i = 4 \ldots N^2-1$ the gauge field configuration in (10) is the SU(N) embedding of the well known SU(2) 't Hooft-Polyakov instanton [3].

In order to extract long range correlations between instantons, it is sufficient to approximate the field strength $F^{(2)\,a}_{\mu\nu}(x)$ of two instantons separated by the distance r by the superposition of the respective field strengths of two individual instantons, i.e.

$$F^{(2)\,a}_{\mu\nu}(x) \;=\; H^a_k \eta^k_{\mu\nu}\, \psi(x^2) \;+\; G^a_i \eta^i_{\mu\nu}\, \psi\left((x-r)^2\right) \, , \tag{13}$$

where the $H_k = t^a H^a_k$ also represent an SU(2) algebra (12). For an instanton anti-instanton configuration the $\eta^k_{\mu\nu}$ of the first term on the right hand side of (13) is replaced by $\bar\eta^k_{\mu\nu}$.

The effective potential only depends on F^2. For the two instanton configuration this quantity can be expressed as (by using $\eta^i_{\mu\nu}\eta^k_{\mu\nu} = \delta_{ik}$)

$$F^{(2)\,a}_{\mu\nu}(x) F^{(2)\,a}_{\mu\nu}(x) \;=\; 3n_H\, \psi^2(x^2) + 3n_G\, \psi^2\left((x-r)^2\right) + 2\, H^a_i G^a_i\, \psi(x^2)\psi\left((x-r)^2\right) \, , \tag{14}$$

where

$$n_H \;=\; \frac{1}{3} H^a_i H^a_i \, , \qquad n_G \;=\; \frac{1}{3} G^a_i G^a_i \, . \tag{15}$$

are the winding numbers of the two instantons. The overlap term in (14) depends on the relative orientations of the two instantons $H^a_i G^a_i$. For an instanton anti-instanton configuration this overlap term is missing (since $\bar\eta^i_{\mu\nu}\eta^k_{\mu\nu} = 0$) and hence

$$F^{(2)\,a}_{\mu\nu}(x) F^{(2)\,a}_{\mu\nu}(x) \;=\; 3n_H\, \psi^2(x^2) \;+\; 3n_G\, \psi^2\left((x-r)^2\right) \tag{16}$$

does not depend on the relative orientation of the instanton and the anti-instanton. For a study of the instanton instanton interaction we first consider the simplest case of a SU(2) gauge group where the instanton (10) coincides with the familiar instanton. For one of these instantons, we can choose $H_i^a = \delta_{ai}$. The 3×3 matrix G_i^a then defines the orientation of the instanton located at $x = r$ with respect to the one at the origin and (14) becomes

$$F_{\mu\nu}^{(2)\,k}(x) F_{\mu\nu}^{(2)\,k}(x) \;=\; 3\,\psi^2(x^2) \;+\; 3\,\psi^2\left((x-r)^2\right) \;+\; 2\,G_k^k\,\psi(x^2)\psi\left((x-r)^2\right) \;. \quad (17)$$

The matrices G_k, $k = 1 \ldots 3$ satisfying (12) can be parametrised as $G_k = U t^k U^\dagger$ with the t^k being the generators of the SU(2) group and U being an element of SU(2). Expressing the unitary matrix U by three angles θ^l, $U := \exp\{i\theta^l t^l\}$ the matrix G_i^k takes the form of an SU(2) Wigner function

$$G_i^k \;=\; 2\,\mathrm{tr}\left(t_k U t_i U^\dagger\right) \;=\; \exp\{-\epsilon_{kli}\theta^l\} \;. \quad (18)$$

The trace of matrix G_i^k entering (17) can be straightforwardly calculated, i.e.

$$\kappa^{(2)} \;:=\; G_k^k \;=\; 1 \;+\; 2\,\cos\left(\sqrt{\theta^k\theta^k}\right) \quad (19)$$

implying that the effective action of the field strength for two fixed instantons only depends on $\theta^k\theta^k$ in the case of SU(2). Note that due to our ansatz (13) the interaction of an instanton anti-instanton pair $\Gamma_{\bar{I}}$ is related to that of two instantons Γ_I by $\Gamma_{\bar{I}} = \Gamma_I(\kappa = 0)$.

In the case of an SU(3) gauge group an instanton which is not an trivial embedding of the 't Hooft Polykov instanton, but has non-trivial colour and Lorentz structure was given in [14]. It has winding number $n_G = 4$ and the corresponding matrices G_i in (12) form the spin one representation. Its colour components in (10) are

$$G_1^7 \;=\; 2 \;, \qquad G_2^5 \;=\; -2 \;, \qquad G_3^2 \;=\; 2 \;. \quad (20)$$

In the following we study their interaction induced by the effective action (8).

The orientation H_i^a of the second instanton is described analogously to the SU(2) case by eight angles θ^a defining the gauge rotation in SU(3), i.e.

$$H_i^a \;=\; 2\,\mathrm{tr}\{U t^a U^\dagger t^b\}\,G_i^b \;, \qquad U \;=\; \exp\{i\theta^a t^a\} \;. \quad (21)$$

In order to calculate the coefficient (relative orientation) $\kappa^{(3)} := H_i^a G_i^a$ of the interference term in (14) we exploit

$$\sum_l (G_l)_{rs}(G_l)_{ik} \;=\; \sum_{a=2,5,7} t^a_{rs} t^a_{ik} \;=\; \frac{1}{4}\,(\delta_{rk}\delta_{is} \;-\; \delta_{ri}\delta_{sk})$$

which holds since the G_l span here the spin one representation. We therefore obtain

$$\begin{aligned} \kappa^{(3)} &= 2\,\mathrm{tr}U\,\mathrm{tr}U^\dagger \;-\; 2\,\mathrm{tr}UU^* \\ &= 4\left[\cos(\lambda_1-\lambda_2)+\cos(\lambda_1-\lambda_3)+\cos(\lambda_2-\lambda_3)\right]\;, \end{aligned} \tag{22}$$

which is expressed in terms of the three eigenvalues λ_i of the hermitean matrix $\theta^a t^a$. The instanton interaction $\Gamma_{I/\bar{I}}$ as defined in (9) was investigated numerically. Figure 1 shows $\Gamma_{I/\bar{I}}$ as function of the instanton distance r for various instanton sizes and for uniquely oriented instantons ($\theta^k = 0$ in (19)). For all instanton radii ρ the interaction is medium range attractive. For instanton radii small compared with scale set by the gluon condensate the interaction becomes short range repulsive. For large size instantons the binary interaction of instantons with same orientation is purely attractive. One should note, however, that due to the use of (13) the estimate of the instanton interaction becomes unreliable for instanton separations as small as the instanton radius.
In the case of SU(2) Figure 2 shows Γ_I for several instanton orientations $\kappa^{(2)}$, which range from -1 to 3. The lowest action is obtained for uniquely oriented instantons ($\kappa^{(2)} = 3$), whereas for maximal disoriented ones ($\kappa^{(2)} = -1$) the instanton interaction becomes purely repulsive. The same is also true for the instanton anti-instanton interaction ($\kappa = 0$).

4. Discussions and conclusions

The exact effective potential of F^2 is mainly determined by two phenomenological ingredients. First, anomalous breaking of scale invariance occurs, which implies that the effective potential is minimal for a non-vanishing value of F^2. Second, the perturbative vacuum is unstable implying that at $F^2 = 0$ the effective potential is a decreasing function of F^2. Therefore the effective potential is expected to have the qualitative behaviour described by the potential V in (7). Let us now assume that instantons are the dominant field configurations contributing to the QCD functional integral. In this case the ensemble of interacting instantons should produce an effective potential which qualitatively behaves like V in (7). With this assumption we may interpret the binary instanton interaction studied in the present letter as the interaction of two instantons moving in the mean field of the instanton ensemble.
Most striking feature of the interaction of two equally oriented instantons is a medium range attractive force. We argue that it is solely due to the instability of the perturbative vacuum. First note that for widely separated instantons (with field strength $F_{(1)}$ and $F_{(2)}$) the main contribution to the integral $\int d^4x\, V\left((F_{(1)}+F_{(2)})^2\right)$ stems from the space-time region near the centers of the instantons. Since for widely separated instantons the field strength of the first instanton is small at the center

of the second instanton, we may expand the above integral and obtain

$$\begin{aligned}
\Gamma_I &\approx 2 \int_{(1)} d^4x \, V'\left(F^2_{(1)}\right) \, F_{(1)} \cdot F_{(2)} \; + \; \int_{(2)} d^4x \, V'\left(F^2_{(2)}\right) \, F_{(2)} \cdot F_{(1)} \\
&= 4 \int_{(1)} d^4x \, V'\left(F^2_{(1)}\right) \, F_{(1)} \cdot F_{(2)} \; , \qquad (23)
\end{aligned}$$

where $V'(F^2) := \frac{d}{dF^2} V(F^2)$. Thereby the space time region (1), where $F^2_{(1)}$ has its maximum, is separated from the region (2) (containing the center of the instanton (2)) by the hyperplane defined by $F^2_{(1)}(x) = F^2_{(2)}(x)$. Figure 3 compares the approximation (23) with the exact numerical result for the effective potential Γ_I. The asymptotic behaviour ($r \to \infty$) of the effective potential is correctly described by (23). Since $F^2_{(2)}$ is a smooth function in the space time region (1), the dominant contribution in (23) stems from the region where either $F_{(1)}$ or V' is large. At the center of instanton (1) the integrand is $V'(F^2_{(1)}(0)) \, F_{(1)}(0) \cdot F_{(1)}(r)$ and therefore of order $\mathcal{O}\left(F_{(1)}(r)\right)$. On the other hand $V'(F^2)$ diverges for small F^2 as suggested by the perturbation theory. Note that perturbation theory, based on a trivial vacuum, is expected to yield reliable results for the effective potential at small F^2. We therefore can rely on the result (7) at small F^2 implying $V'(F^2) = \ln F^2/F_0^2$. At the hyperplane the integrand in (23) is therefore of order $\mathcal{O}\left(\ln[F^2_{(1)}(r)] \; F_{(1)}(r)\right)$. This implies that behaviour of $V'(F^2)$ in (23) near the perturbative vacuum ($F^2 = 0$) is relevant, where we have $V' < 0$ due to the instability of the trivial vacuum. Note further that for equally oriented instantons $F_{(1)} \cdot F_{(2)}$ is positive, and we end up with a medium range attractive instanton instanton interaction.

In conclusion, we have investigated the binary interaction of instantons with size large compared with the scale provided by the gluon condensate. This interaction is derived from the effective action for F^2. The effective potential at small field strength, which is accurately estimated by the leading order of the loop expansion, is relevant for the instanton interaction at medium range. In order to estimate the short range interaction of instantons, the effective potential is also required for large fields, which is beyond the validity of the loop expansion. For large fields we modelled an effective potential consistent with renormalisation group arguments and phenomenological requirements.

We find that the interaction of two uniquely oriented instantons with radii large compared with the gluon condensate is purely attractive. Equally oriented instantons have minimal effective action. Since in this case no repulsive core prevents the large size instantons from collapsing, this result might indicate a condensation of large scale instantons forming a homogeneous gluon condensate. The instanton anti-instanton interaction induced by the scale anomaly is repulsive for all instanton orientations. We note, that our results for the short range instanton interaction depend on our model assumptions of the effective potential at large field strength and are strictly valid only for large instantons separations.

The situation can be compared with that of an Heisenberg spin system, where ferromagnetism is obtained by investigating the binary interaction of spins induced by the mean magnetisation of the spin ensemble. In analogy, it might happen that the instanton interaction accounts for a gluon vacuum which decomposes into domains of a coherent superposition of (anti-) instantons. This picture of the gluonic ground state is consistent with that emerging in the field strength approach to Yang-Mills theories [15, 16]. The precise form of the instanton vacuum of QCD will likely be determined by the interplay between the attractive instanton interaction (favouring a homogeneous instanton condensate) and the entropy (favouring a domain or liquid type vacuum). This is an challenging subject for future work which requires a detailed numerical simulation of the instanton medium [6, 7, 8] using the above extracted instanton interaction as ingredient.

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Figure captions:

Figure 1: The interaction Γ_I of two uniquely oriented instantons as function of the instanton distance r for several instanton radii ρ. r and ρ in units of $(F_0^2)^{-1/4}$. Γ_I in units of the gluon condensate F_0^2.

Figure 2: The (anti-) instanton interaction Γ_I as function of the instanton distance r for several instanton orientations κ and for $\rho = 2$. r and ρ are given in units of $(F_0^2)^{-1/4}$. Γ_I in units of the gluon condensate F_0^2. The interaction of an instanton anti-instanton pair $\Gamma_{\bar{I}}$ is also shown ($\kappa = 0$).

Figure 3: The instanton interaction Γ_I provided by the approximation (23) compared with the exact numerical result for $\kappa = 3$ and $\rho = 2$. r and ρ are given in units of $(F_0^2)^{-1/4}$. Γ_I in units of the gluon condensate F_0^2.